

String Quartet #1

M i c h a e l E d w a r d E d g e r t o n

Dedicated to
And
Written for

the

Kairos Quartett

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Chatschatur Kanajan, vl
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Principles

The influence of nonlinear phenomena and the scaling of multidimensional phase space served as generating principles for musical composition in this **First String Quartet**. As will be shown, two broad applications seem to have had a particularly robust potential for musical expression. The first involves the use of non-linear dynamics to structure large-scale formal development, while the second directly effects local sound production and gesture.

In the early 1980's, theories of nonlinearity were beginning to be applied to complex musical signals that led some to reconceptualize their understanding of the elements involved in the production of sound. For some, this led quite logically to multidimensionality of extra-complex musical sonorities. Most often, this increased awareness has been applied in computer musical contexts. However, as acousticians, programmers, composers and performers had begun to systematically look into the tiny bits of sound, they found that it was possible to pull apart the texture of instrumental and (less often) vocal production. One of the pragmatic ways this was and is done is to look at the elements involved in the production of a sound and to explicitly change certain variables one by one while keeping the others constant.

This **String Quartet #1** is involved with multidimensional networks whose internal variables are shifted within an scalable environment. My use of the term scalable suggests that a variable is assigned a minimal and maximal value. Then between these extremes, more or less discrete values are inserted, so that a sequence of linear steps from low to high, slow to fast, etc. is developed. The value of scaling these parameters affects global compositional ratios of novelty versus redundancy. Significantly, this suggests that the logical procedures of composed sound may stretch across 8 to 10 dimensions, rather than the usual 2 (pitch and rhythm). Then in certain cases, as the parameter space is filled with an increased activity of non-idiomatic behavior, bifurcations may appear to push the output into nonlinear phenomena appearing as unexpected, transient or extra-complex musical sonorities.

Nonlinear Phenomena

Nonlinear phenomena have been reported in many diverse disciplines, including physics, health sciences, engineering, literature, neurology, geology, music, etc. Directly relevant to this composition, nonlinear phenomena have been reported for newborn cries [1], pathological voices [2], extra-normal 'extended' vocal technique [3], animal vocalizations [4], as well as flute [5], oboe [6], saxophone [7], trombone, crumhorn, bassoon [8], trumpet [9], and violin [10].

Analysis of real-world phenomena using methods from nonlinear dynamics are frequently based on descriptions of a system within a phase space. The phase space is built from dynamical variables that are necessary to determine the state of the nonlinear system. At every moment, the behavior of a system may be represented by a single phase space point. It has been found that phenomena frequently reach a particular dynamic regime after initial transients. This regime corresponds to a geometrical object in phase space and is termed an attractor. Four types of attractors have been identified: 1) Steady state, a behavior whose variables are constant; 2) Limit cycle, periodic behavior (repeating itself continuously); 3) Torus, a two-dimensional object in phase space that results from the superposition of two independent oscillations; 4) Chaotic attractor, a nonperiodic behavior that never repeats but stays within a limited space [11].

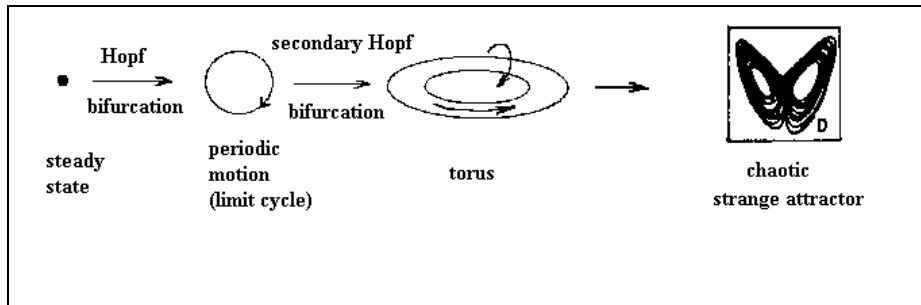


Fig. 1: attractor states with associated bifurcations

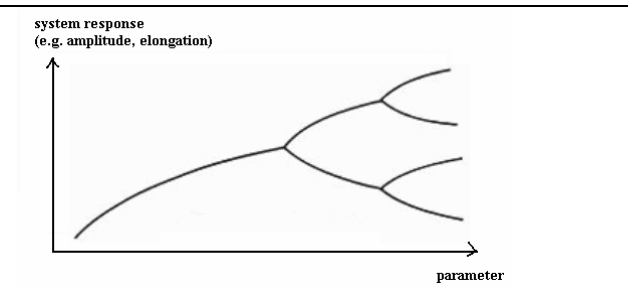


Fig. 2: one dimensional bifurcation diagram

Attractors govern the dynamics for constant external parameters such as vocal fold tension or subglottal pressure, in the case of phonation. Often these parameters vary slowly and may feature sudden transitions to new attractors. These transitions are termed bifurcations and include: 1) Hopf bifurcation, a transition from a steady state to a limit cycle; 2) Period doubling bifurcations, transitions from a limit cycle to folded limit cycles; 3) Secondary Hopf bifurcation, a transition from a limit cycle to a torus. Further, subharmonic bifurcations and tori often are precursors of deterministic chaos, such that small parameter shifts induce jumps to nonperiodic oscillations. A comprehensive visualization of transitions can be achieved by bifurcation diagrams [12] which display different dynamical behavior depending on one or two varying system parameters.

As applied to voice, steady state behavior occurs when the vocal folds are at rest. Then as subglottal air pressure begins to rise a Hopf bifurcation occurs to push the steady state attractor into a limit cycle as the vocal folds begin to produce normal periodic vocal fold vibrations. Often during speech and song, period doubling bifurcations occur and lead to subharmonic oscillation. Subharmonics may be classified as a folded limit cycle, that often appears via transitions from periodic oscillation to an oscillation with alternating amplitudes, or as an addition of a second periodic source, locked at a frequency ratio of 1:2. Less frequent, though still seen in speech and song are phenomena featuring two or more independent frequencies. This phonation, classified as a torus, may be produced with (left-right) asymmetrical vocal fold vibration and has been termed biphonation [13]. As mentioned above, subharmonics and tori often are precursors of deterministic chaos, which includes high airflow multiphonics as an example.

Applications

Fig. 3 shows bifurcation diagrams showing the experimental results from excised Larynge Experiments. Both diagrams show asymmetries of vocal fold adduction as experimentally applied to excised canine larynges. In Fig. 3a, we see the results that an increase or decrease of micrometer asymmetry (x axis) and subglottal pressure (y axis) produces. With low subglottal pressure, an increase of applied asymmetry had no effect on phonation, as it remained within a chest-like vibration. However, as subglottal pressure increases, irregular vibrations (including period three subharmonics, transient Fo) and periodic single vocal fold oscillations were observed. Likewise in Fig. 3b, an increase or decrease of micrometer asymmetries and subglottal pressure led to chest-like vibrations, falsetto-like vibrations, vortex-induced (whistle-like) vibrations and instabilities. For the larynx shown in 3b, it might be interesting to note that an increase in subglottal pressure at low to medium asymmetries did not result in instabilities, but rather, remained in chest-like vibration – however, as might be expected an increase of asymmetry coupled with an increase of subglottal pressure produced instabilities [14].

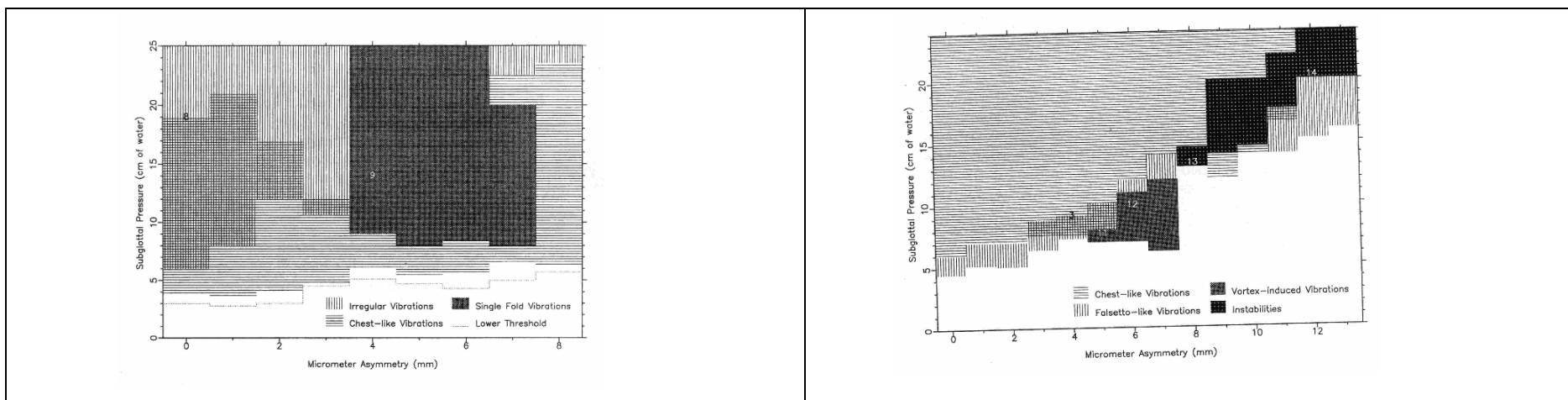


Fig. 3: bifurcation diagrams associated with bifurcations in excised larynge experiments

Closely associated with the well-known butterfly effect, in which small changes of initial conditions may produce large effects in the systems output, these bifurcation diagrams provide experimental evidence that small perturbations may lead to nonlinear results in a musical instrument, the voice. Therefore, the idea of shifting sound production variables in instruments and voices is directly linked to experimental research (as well as to traditional, world music and electroacoustic/computer music experiences). Next, it may be useful to provide a brief description of a solo work for violin (Mamre) which offers a somewhat simple framework from which to view the scaling of the multidimensional parameter space.

Mamre

In MAMRE, a short study for solo violin, a select group of variables were chosen that would offer a closer look into the micro-sound world of the violin. More specifically, the intention was to develop a multi-parameterized network that would allow compositional coherence to be developed across multiple dimensions. Aurally, the result of such a framework resulted in the production of irregular and transient sonorities by shifting inherent variables of sound production into non-idiomatic ratios, when compared with pitch, rhythm and tempo. In addition to rhythm and pitch, the following variables were selected: bow rotation, bow speed, microintervallic movement, microintervallic tuning (Beats), bow placement, bow pressure, decoupling bow speed from tempo, two dimensional vibrati. The results of these manipulations include transient source and spectral segments; complex harmonic and inharmonic multiphonic sonorities; subharmonics, and; inharmonic glissandi. Next, a few spectrographic analyses (with accompanying recordings) may assist in this discussion. The analyses of Mamre were taken from a studio recording by the violinist Chatschatur Kanajan of the Kairos String Quartet [15].

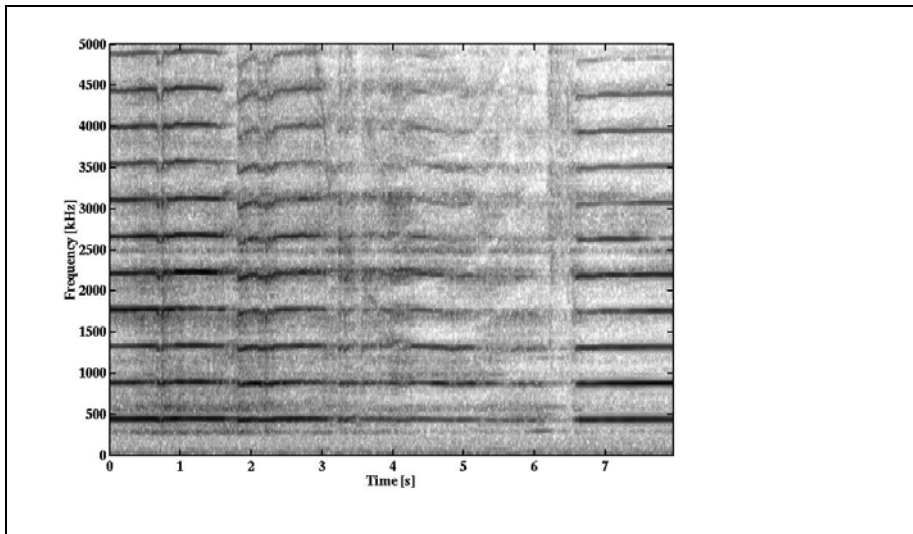


Fig. 4: ord to col legno to ord; $\frac{1}{4}$ tone glissando; bow placement glissando

In Fig. 4, the results of shifting from ord to col legno to ord, with a $\frac{1}{4}$ tone glissando, featuring a glissando of bow placement from normal to tasto 4 (hi over fingerboard) to normal are shown. Beginning with a somewhat normal tone, the image shows a disruption near 2" where the bow is shifted to col legno. Then from approximately 3" to 6.5" a filtering of the spectrum occurs to reduce the amplitude of all harmonics (even the fundamental), and above the fourth harmonic significantly more energy is reduced. The result becomes muffled and transient, completely stopping the tone near the 5-6" timing. As well, an inharmonic glissandi appears to be the result of the motion of the wood sliding up and then down the string.

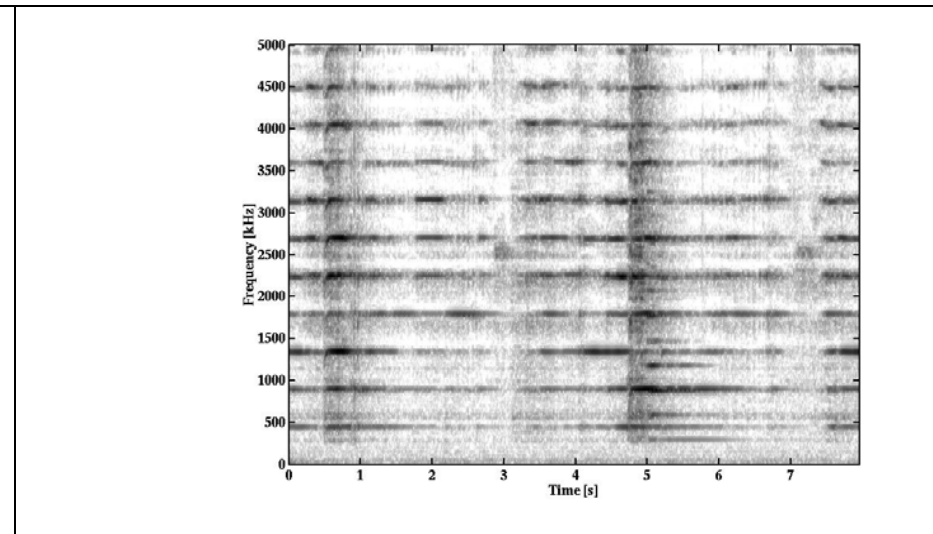


Fig. 5: fast bow speed; change of bow placement

In Fig. 5 an extremely quick bow speed, using approximately $\frac{3}{4}$ of the bow is combined with changes of bow placement. The prominent effect is of spectral transience. At approximately 5 seconds, an inharmonic band is produced through heavy bow pressure, slightly sounding the d-string.

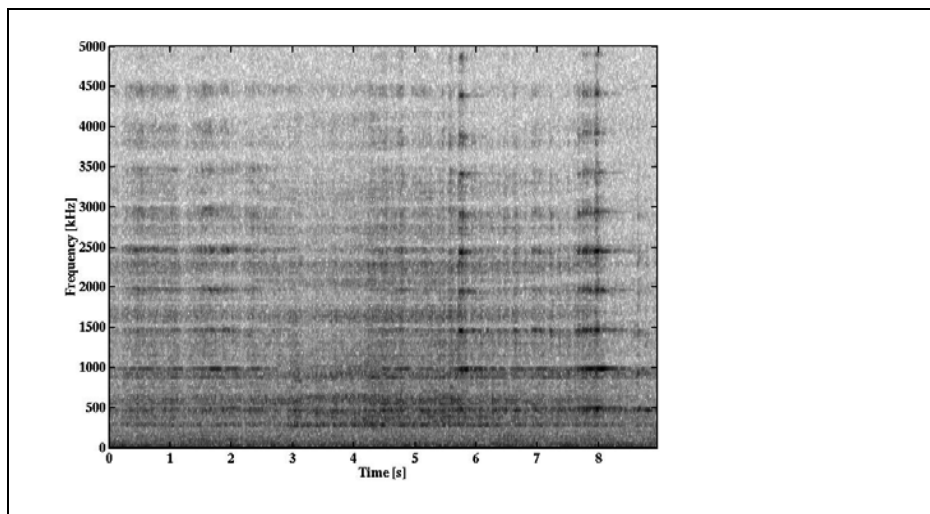


Fig. 6: wood alone; bow placement from ponticello 2 to bridge

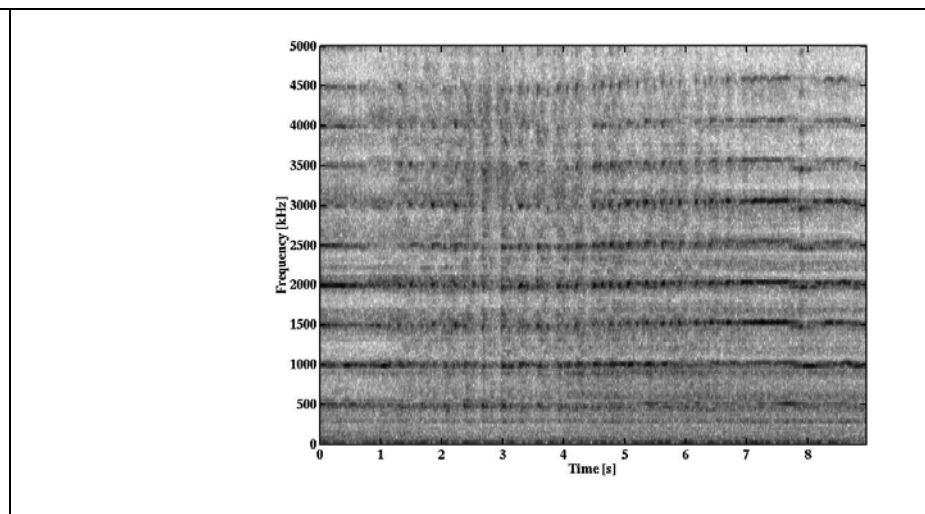


Fig. 7: slow (as slow as possible) bow speed; oscillation of bow placement between ponticello 1 and ponticello 2; high to maximum microtonal pitch detuning between adjacent strings (d, a)

Fig. 6 features a sequence in which only wood is used to produce the source sound, while the bow placement switches from ponticello 2 (next to the bridge) to directly on the bridge. The resultant sound is a complex tone of harmonic and inharmonic components. Note how the spectral components shift over time. From 0-2", the harmonic energy is clearly defined, then from 2-4.5" the harmonic energy is significantly reduced. From 4.5-9" the spectrum features bursts of harmonic energy, with loud bursts at 5.8" and 8".

Fig. 7 features a slow bow speed, with an oscillation of bow placement between ponticello 1 and ponticello 2 and high microtonal pitch detuning between the d and a strings. Note the rhythmic disruption to the spectra caused by the sliding hairs of the bow and the extremely slow bow speed, that serve not only as an almost percussive event, but also to filter out particular frequencies. Note as well as the relative strength of harmonic energy from 4.5-8" that results from the placement of the bow so near the bridge.

MAMRE, influenced by the performance practice of an al-rabab (a two-stringed Egyptian dichord), begins to explore the micro-sound world through perturbations within the multidimensional parameter space. The results are positive and directly mirror observations of our real, physical and nonlinear world through irregular, transient and non-stable phenomena.

String Quartet #1

The **String Quartet #1** is the logical extension of the multi-dimensional network used in the solo piece Mamre that features my fullest treatment of the global and local applications of nonlinear dynamics in my work.

As was seen in MAMRE, the result of shifting variables produced extra-complex musical structures featuring a crucial property of transition and change. Here, in the String Quartet #1, it might be readily apparent that simultaneously shifting multiple variables (including: bow rotation; bow angle; bow portion; bow length; effleure (left hand pressure); bow speed; pitch; rhythm; intensity; placement; bow attack and release) induce increased bifurcations that impact the resultant sound in often deterministically nonlinear ways.

Formally, movement one references sonata form (without its tonal implications) and pretonal contrapuntal complexes. Nonlinear dynamics becomes part of the functional development through tempi relations that are formed from the fractal dimension found within the outline of the Koch curve (in which an infinite length fits within a finite area, such as may be found in a coastal outline), at an increase of 1.2618. Meanwhile, the treatment of multi-dimensional frameworks is scalable and complete.

Movement two is formally a musical riddle, influenced by the composition UT, RE, MI, FA, SOL, LA by the renaissance composer John Bull, in which the procedure of transcribing letters to integers serves as a formal, generative process [16]. As well, the style of this movement references a typical slow second movement, but significantly diverges to other regimes as the multi-parameterization assumes a heightened characteristic.

Movement three presents a view of the concept of divergence through the decoupling of bow speed and portion when compared with tempi. In practice, this is a highly nontrivial situation, as string players are taught to correspond bow speed with tempi. Even slight deviations from this norm often result in non-musical performance and in this string quartet present significant psycho-physiological hurdles to navigate around. In order to explicitly heighten this decoupling, easily identifiable gestures were chosen, that are to be performed at extremely quick tempi. The resultant effect should be dramatic, and if truly dedicated, will feature unstable, transient episodes.

Movement Four is based upon the unfolding within phase space of a strange attractor (a behavior that is stable and non-periodic, that stays within a definable phase space, yet never quite repeating). For the fourth movement, the image of a spiral confined by a box, infinitely deep and not quite repeating became the organizational principle. The process involved abstracting a few geometrical shapes that were subsequently unfolded, so as to fit within a two-dimensional graph, identified as pitch versus time. The effect is one of sliding glissandi, which are then radically shifted according to the results of multidimensional scaling.

Movement five is the most complete treatment in the application of nonlinear dynamical thinking to form and production. In this movement I translate the logistic equation (a model of long-term population change) onto the multidimensional parameter space (see figure 2). When the control parameter is less than 1, all iterated values decay to zero (meaning that a particular population becomes extinct). Then as the control parameter increases between 1 and 3, iterations converge to a single value. Between 3.0 and approximately 3.58, a series of bifurcations occur that are highly sensitive to the value of the control parameter. First it converges to two final values, then continuing to 3.5, four values result, and continue to increase until approximately 3.58, in which chaos appears. Between 3.58 and 3.99, the behavior is not purely chaotic, as windows of periodicity occur within this information-rich field [17]. As the period-doubling, -tripling, and -quadrupling is musically limited, I decided to focus on the deeper levels within the cascading series at values above 3.57. This governs localized form. Globally, the fifth movement is governed by the Mandelbrot set, in which large active regions are joined by thin, filament-like strands to far-reaching islands. The metaphor provided by the long filament-like strands provided the right cues for developing purely inharmonic and low-amplitude sections.


To end with a quote from James Gleick, “Equally critical in biological systems is flexibility: how well can a system function over a range of frequencies. A locking-in to a single mode can be enslavement, preventing a system from adapting to change. Organisms must respond to circumstances that vary rapidly and unpredictably; no heartbeat or respiratory rhythm can be locked into the strict periodicities of the simplest physical models ... Goldberger noted “fractal processes associated with scaled, broadband spectra are ‘information-rich’. Periodic states, in contrast, reflect narrow-band spectra and are defined by monotonous, repetitive sequences, depleted of information content.” Treating such disorders, he and other physiologists suggested, may depend on broadening a system’s spectral reserve, its ability to range over many different frequencies without falling into a locked periodic channel.” Then quoting Mandell, “is it possible that mathematical pathology, is chaos, is health? And that mathematical health, which is the predictability and differentiability of this kind of structure, is disease [18]?”


References


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STRING QUARTET NO. 1

Performance Notes

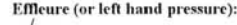
Bow rotation: 


Bow length and bow angle: 


Bow speed: 

Pitch, rhythm: 

Bow placement: 

Effleure (or left hand pressure): 

Bow Attack: 

Bow Release: 

BOW ROTATION: full hair



slight hair



wood and hair



wood only



BOW LENGTH (+ portion) AND BOW ANGLE:































Bow length and portion:

10 bow lengths are identified relative to a three-part division of the bow: tip, mid, frog;

Within each field (or graph, shown at far right), two complementary elements are identified: bow length and bow angle. These are identified in two ways: 1. through a graphic notation that displays the approximate length/portion and angle to be used, and; 2. through the paired letter and number notation found at the top of each field.

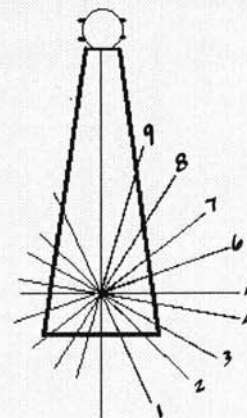
Each letter represents a length and portion of the bow to be used and are identified to the right. All of the lengths and portions will feature slight variations in performance.

The following integer represents the angle of the bow relative to the fingerboard on the horizontal plane.

Tip	mid	Frog	
			A = 1/3 mid
			B = 1/3 tip
			C = 1/3 frog
			D = 2/3 mid + tip
			E = 2/3 mid + frog
			F = 2/3 center
			G = full bow
			H = slight mmt mid
			I = slight mmt tip
			J = slight mmt frog

BOW ANGLE:

11 bow angles are identified, relative to fingerboard;



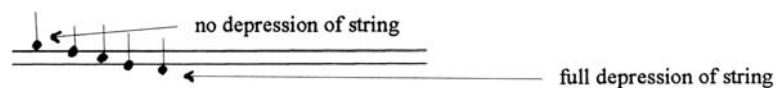
+90° = 10
+72° = 9
+54° = 8
+36° = 7
+18° = 6
Normal = 5
-18° = 4
-36° = 3
-54° = 2
-72° = 1
-90° = 0

COMBINED: length and angle

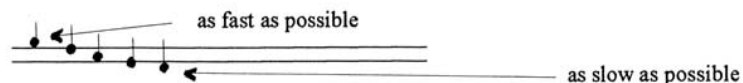
C.7

= 1/3 frog at 36°

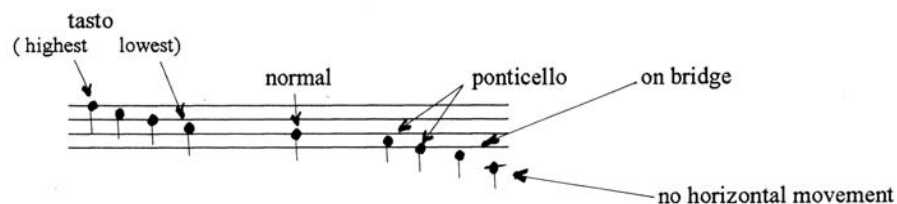
Effleure:



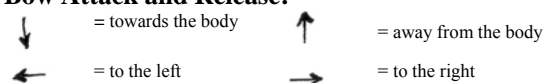
Bow Speed:



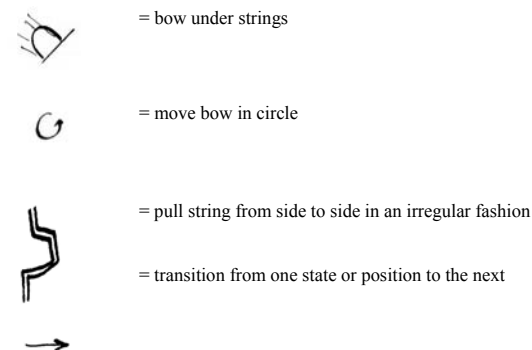
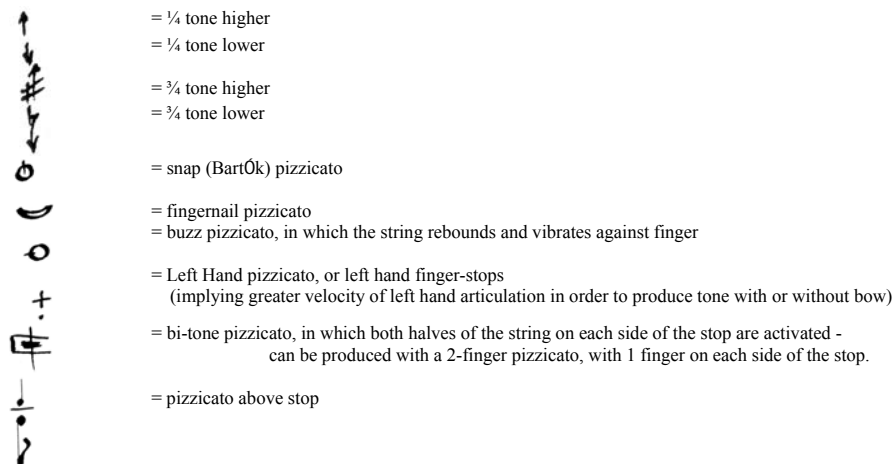
Bow Placement:



Bow Attack and Release:

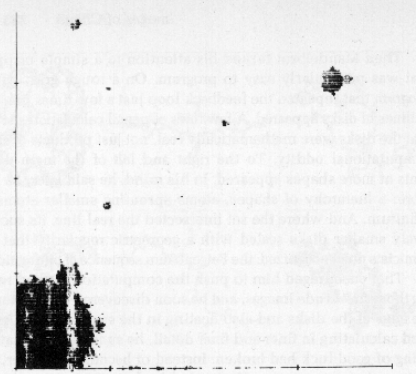


General Indications



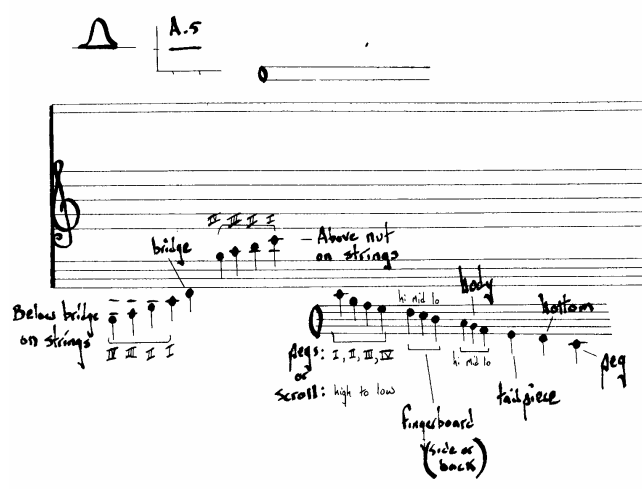
Specific indications for Movement Five

Further descriptions

	<p>As indicated above, one of the early computer printouts produced by Benoit Mandelbrot metaphorically governed the formal construction of the fifth movement. This graphic is shown to the left below.</p> <p>Simultaneously, I wanted to achieve two complementary compositional aims: 1) to synthesize and reinterpret material from the first four movements, and; 2) to introduce new material. These aims, when linked with the Mandelbrot graphic, were interpreted in the following way.</p> <p>The islands of visual activity seemed to provide an appropriate landscape in which to reassemble the sonically active material from the first four movements. As Mandelbrot and other mathematicians found, when the quality of computation improved, each level of magnification roughly followed the principle of self-similarity at different scales, but showed that none of the successive molecules exactly matched one another – there were always new species appearing. Musically, this rough resemblance, implicating the dynamic process of change and a broadening of the fractal concept, served as a guide to the reinterpretation of the materials from the first four movements. Specifically, the elements reappeared along a continuum from roughly similar to dramatically altered in gesture, technique and expression.</p> <p>Secondly, the islands of visual activity seem at first glance to be spatially isolated, but a new mathematical approach (Douady and Hubbard) proved that the outlying dust hangs on a delicate web connected to the main body. This proved that all dust, no matter how small, would roughly resemble the main set, each with a unique character. For my purposes, the suggestions of a fine, delicate and unseen web served the intention of exploring primarily soft, inharmonic sections of activity. Therefore, I applied the previous multidimensional conception to sound production extending beyond the normal nut-to-bridge bowed/plucked string sound. These inharmonic sonorities were somewhat strictly governed by the logistic equation as described on page vii. By carrying the calculations to 7 decimal places behind the prime, the resultant integers were then applied to a network of over 80 scalable classes of variables. The result was to focus on the deeply-hidden windows of periodicity found within complex structures.</p>
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Production

In an attempt to address scalability within primarily inharmonically sound production, the multidimensional framework found on **page ix** was expanded to include new regions of production and to offer a dual functionality of the previous notational devices.

	<ul style="list-style-type: none"> = bow rotation, bow angle and portion, and effleure are the same as found during the first four movements = bow speed as found during the first four movements = pitch and rhythm as normal = Placement on strings – in the fifth movement placement <i>above nut</i>, <i>below bridge</i>, and <i>between nut and bridge</i> acquire a heightened functionality similar to the frequency characteristic of ordinario pitch production (between nut and bridge) as notated on the five-staff stave. = On body (regions other than strings)
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In the score, the following symbols are used:

FR = front-right; FC = front-center; FL = front-left; BR = back-right; BC = back-center; BL = back-left;
F = front; B = back; C = cranks (or cloth on string below bridge); S = side

- = damping strings
- = holding/gripping bow with both hands, so that power source (bow hair) is shortened
- = approximate placement of hands on bow
- = pressure on bow pushing hair against wood
- = on or with the cranks
- = turning cranks; ↑ = up in pitch; ↓ = down in pitch
- = bow under string
- = bow under string, on bridge
- = bow under string, on bridge with bow hairs flat against bridge
- = bow under string, on bridge with wood against bridge
- = bow under string, on bridge with bow hair against bridge, wood against string(s)

positions of bow on bridge

- = on top
- = off-center, near edge
- = on edge
- = on side of bridge
- = bow on bridge side at an angle
- = bow on bridge side, perpendicular to bridge
- = bow in crook of neck and body on back

- = with tension screw
- = pulling string to one side or side-to-side, as indicated in score
- = music stand (used to indicate stand strikes with inexpensive bow)
- = with tuning fork
- = with thimble (preferably metal)
- = regular tremolo; irregular tremolo
- = loose ends of strings
- = peg 1 inside peg box;
-  = peg 2 inside peg box
- = soft rubber mallet
- = onset to offset characteristic
- = bow above stop
- = tremolo or oscillate between wood walls of inner scroll box
- = rotate bow (used primarily for glass bow)
- = metal plectrum

bows

normal, glass, wood (with notches), plastic (smooth), plastic (with notches), inexpensive
ord., ord. with coarse hair, ord. with loose hair, metal

manner

bowing, pluck, flick, rub, left hand alone, bow with two hands, battuto (strike, tap)

Specific Indications (correspond to markings in score)

movement one

- *1 Mostly artificial harmonics occur at the 4th. Occasionally, artificial harmonics other than the fourth are intended, and are duly marked with the appropriate integer, parenthetically, over the notated fundamental and point of articulation. However, other transient artificial harmonics are identified as slight alterations from the P4th, P5th, M3rd, m3rd and so on. These alterations should result in fleeting sonorities. (p.1.)
- *2 A smooth, quick transition from F.4 to F.6 (or other angles and lengths, as notated), emphasizing the transition. The 2nd state should be treated as an arrival. (p. 2, 70, 72, etc.)
- *3 two-finger pizzicato on adjacent strings (p. 3.)
- *4 three finger pizzicato (p. 5.)
- *5 “alla flamenco” strum violin like a guitar in the ethnic flamenco style – take care to alter speed and dynamic of strum. (p. 5, 6, 7, 9, etc)
- *6 “alla zingarese”, left-hand pizzicato tremolo, while bowing on the strings as indicated – the transitions between left-hand pizzicati should be captured with dynamic bow changes, resulting in somewhat transient ratios between arco and pizz – like gypsy music, it should be highly rhythmic and intense (although not necessarily loud!). (p. 5, 9, etc)
- *7 col legno battuto (beat string with bow) (p. 6)
- *8 Pizzicato on notated string with left hand above the stop indicated. (p. 7, 11, 14, etc.)
- *9 High, natural harmonics are intended to have slightly transient properties, especially when combined with shifting multidimensional variables. (p. 8, 34, etc.)
- *10 left hand finger-stop pizzicato tremolo (p. 13)
- *11 left hand pizzicato above stop (p. 13, 14, 23, etc.)
- *12 left hand pizzicato, while striking string (battuto) with wood only bow (p. 14)
- *13 left hand finger-stop pizzicato, while striking string (battuto) with full hair bow (p. 15, 20)
- *14 left hand trill combined with right hand tremolo (p. 15)
- *15 subharmonics. (p. 15, 61, 67-69, etc.)

Subharmonics, consisting of periodic sounds with a pitch lower than the string fundamental frequency, are representative of a special type of nonlinear attractor state – that of a period doubling or folded limit cycle. These may be produced with high bow pressure, slower bow speed and careful bow placement. A subharmonic series carries an inverse relationship to the related harmonic series, such that instead of producing a whole-number multiplication of the fundamental, a division of the fundamental occurs. As is shown below, the procedure for producing subharmonics involves the stopping a pitch (or open string) with the left hand while, bowing on a node of the (sub-) harmonic at which the bow is placed, in order to produce the resultant subharmonic pitch. Additionally shown below is the (sub-)harmonic series over a g#. Theoretically, all (sub-)harmonics should be available on all nodal points along the string. However, practice indicates that certain nodal points will respond better on different instruments – so the suggestion is to search for those nodal points that respond best.

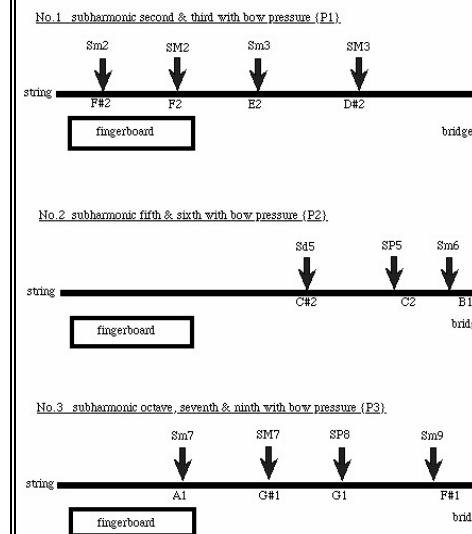
Note that the fifth movement expands upon the diversity of (sub-)harmonics to be produced, and therefore the notation is further developed. In this fifth movement, subharmonics show the stop pitch, the harmonic upon which the bow is to be placed (exact nodal point to be decided by performer) and the resultant pitch identified by the number of the subharmonic, found under the notated pitch – this number should carry an inverse relationship to the harmonic node. In mmt 5, mms 112-222, ord. harmonics, which are identified with a nodal point and a number above the resultant pitch, alternate with subharmonics.

A physical description of subharmonics has been provided by Fletcher and Rossing, as follows “In the Helmholtz model for normal bowing, the traveling kink, which has a period the same as that of a freely vibrating string, serves as a synchronizing signal to ensure that the period for the slip-stick process is the same as the natural vibrational period of the string. If the bow force is great enough, however, the maximum transverse frictional force exerted by the bow on the string is sufficient to prevent the Helmholtz kink from triggering the release of the

string from the bow hair, and the bow continues in the sticking part of the cycle until some other small signal triggers the slipping process. If careful bow control is exercised, it is possible for a periodic motion with longer than the normal period to occur as a result of regular triggering by the same part of the waveform for each cycle. This can result from the combined effects of multiple bow-nut and bow-bridge reflections of transverse waves and reflections of torsional waves (Hanson et al., 1994; Guettler, 1994). “ (From: The Physics of Musical Instruments by Neville H. Fletcher and Thomas D. Rossing, Springer-Verlag: New York, 1998.)

To end this discussion of subharmonics is the following information taken from a paper titled “Subharmonics: A Revolutionary Technique for the Violin” by M. Kimura, published in ASVA 97 (International Symposium on Simulation, Visualization and Auralization for Acoustic Research and Education) Proceedings.

[Fig. 1] Bow location for subharmonics on open G string (G2)



“Each interval such as second, third, fifth and octave below the fundamental pitch, requires different speed, pressure, and location on the string. One can isolate different subharmonics by normalizing the speed and pressure of the bow accordingly, precise control achieves regular, repeatable, and dependable results.

some subharmonic intervals are obtained by exerting the same amount of bow pressure. The only notable element that separates these intervals is the slight shift in the emplacement of the bow on the string. [fig. 1] shows the relative bow location on the string for playing different subharmonics on open g string. For example, subharmonic minor second (sm2), major second (sm2), minor third (sm3) and major third (sm3) are obtained by using almost identical bow pressure and speed, which is called {p1} (see [fig. 1] no. 1). Similarly, the bow pressure {p2} is identical for the subharmonic perfect fifth (sp5), minor sixth (sm6), diminished fifth (sd5) (see [fig. 1] no. 2), and so is the bow pressure {p3} for subharmonic octave (s8), minor seventh (sm7) and major seventh (sm7) (see [fig. 1] no. 3). Larger intervals such as ninth, eleventh (octave & third), and thirteenth (octave & fifth), can be obtained in a similar manner.

there are several variable elements that are included in order to obtain subharmonics: the amount of rosin, the age and thickness of the string, and the composition of the string. These variables are often combined and create technical problems, mainly affecting the location of the bow on the string. The amount of bow pressure is the most crucial element, however, which one must imagine precisely before playing subharmonics.”

- *16 left hand thumb pizzicato (p. 16)
- *17 left hand pizzicato, while right hand produces pizzicato above stop (p. 17)
- *18 left hand oscillation between two chords, while right hand produces a tremolo (p. 21)
- *19 a continuous and deliberate transition between the two states (here F8 to f3). (p. 21, 22, etc)
- *20 left hand pizzicato on an open string, while right hand produces tremolo on stopped string. (p. 25)
- *21 battuto buzz – wood only, similar to buzz pizzicato, in which the string rebounds and vibrates against finger, except here the string vibrates against the wood of the bow. (p. 25)
- *22 left hand pizzicato above the nut, while right hand produces pizzicato (p. 25)
- *23 play with guitar pick (p. 26)
- *24 slap pizzicato – slap strings with right hand (p. 27)
- *25 battuto bow pizzicato - striking string (battuto) with full hair bow (p. 27)
- *26 effleure – transient and shimmering effects, always gentle (p. 28-29)

movement two

- *27 the second movement uses tunings that differ from equal temperament to facilitate unique pitch correspondences, especially when performing mid- to high-natural harmonics – see page xiii for chart indicating frequency alignment in herz and cents. (p. 30 onward)
- *28 altered artificial harmonics – meaning the stop and slight finger depression at a fourth results in harmonic 4; while a 5th results in h° 3; a M3rd results in h° 5, and; a m3rd results in h° 6. Then added to these, extra alterations are introduced in order to produce bifurcations to transient and perhaps even folded limit cycles, toroidal and strange attractor states. See note #1. (p. 36, 37, etc.)

<p>*29 the ledger line below the two-line bow speed stave indicates no horizontal movement – in this case, the bow travels from ponticello 2 to placement as high as possible over the fingerboard. (p. 36, 37, etc.)</p> <p>*30 the double-headed arrow (arrow on each end) with one, two or three stems refers to slow, medium, and fast oscillations between notated states – in this case an oscillation of bow placement. (p. 37)</p> <p>*31 still no horizontal movement, only an oscillation between ponticello 2 and as near as possible to stop – in this case the natural harmonics. Add to this mix the sustained, then staccato stops on the a-string. (p. 38)</p> <p>*32 gettato – bounce bow in a single stroke on string while executing a glissando in the left hand. (p. 39, 41)</p> <p>*33 ad lib harmonics. (p. 40)</p> <p>*34 the plus or minus values refer to approximate absolute values in cents of raising or lowering both the stop and the harmonic articulation. (p. 42-44)</p> <p>*35 play melodic sequences on natural harmonics within the specified tessitura that more-or-less follow the contour notation. (p. 45)</p> <p>*36 degrees of tuning and detuning beats of the 8ve, with 0 being a pure octave and 10 being the most beating (not the furthest detuned!). (p. 45, 46)</p> <p>movement three</p> <p>*37 whatever tempo is chosen, it should remain resolute and return during similarly labelled successive sections. (p. 47)</p> <p>*38 oscillation between identified states, in this case between bow length and angle positions. The speed of oscillation is identified by the double-headed arrow with either one, two or three stems, indicating slow, moderate or fast rates of oscillation. (p.1, 48, 66)</p> <p>*39 oscillation between states that feature a transition from fast to slow rates of tremolo (or vice-versa). (p. 48)</p> <p>*40 glissando on harmonics as identified. (p. 48)</p> <p>*41 slap strings with left hand on e-string to produce a broken glissandi as the stops will move progressively closer to the open string after the initial downbow. (p. 53)</p> <p>*42 left hand finger-stops produced with greater velocity – without bow – a continuation of gesture. (p. 55, 56, 64)</p> <p>*43 place tuning fork on body of cello at a location to maximize the resonant coupling. (p. 57, 58)</p> <p>*44 play resolutely legato until mm. 133. Phrase each line according to bow changes. As each instrument has a different ratio of tempo and bow speed, the phrasings should be dramatically different. (p. 58)</p> <p>*45 repeat pattern until otherwise indicated. (p. 59)</p> <p>*46 glissandi on c-string with a broken glissandi on g-string. The broken glissando is to be performed with left-hand finger stops, which should carry the gesture of a series of grace-notes. (p. 63)</p> <p>*47 effleure, positions 3 to 4 – transient, ghost-like return. (p. 64-65)</p> <p>movement four</p> <p>*48 a quick transition between the first and second state. (p. 71)</p> <p>*49 oscillation between a full stop and position 4 effleure. (p. 72)</p> <p>movement five</p> <p>*50 stops produced by depressing string with fingernails. (p. 77)</p> <p>*51 switch to flesh of fingertips. (p. 78)</p> <p>*52 double pizzicato glissando on one string. Left-hand finger produces glissando as indicated, fingers 1 + 2 of right hand straddle left-hand finger, and both produce a pizzicato. Rearticulate right-hand pizzicati until left-hand reaches the indicated end-note of glissano. In effect, a continuous double glissando produced by multiple pizz. source. (p. 82)</p> <p>*53 arco rub between positions as indicated, on string indicated. (p. 84)</p> <p>*54 chin pulses – gentle pulsation of instrument with chin. (p. 84)</p> <p>*55 hold against torso – gently and firmly as if instrument were a baby – but NOT in a broad theatrical gesture. (p. 84)</p> <p>*56 shifting from hi to lo position on tailpiece. (p. 84)</p> <p>*57 oscillate between sul III and IV, while pulling string side-to-side. (p. 85)</p> <p>*58 detune string a whole step, then stop pitch one ½ step above Fundamental, in order to produce effleure. (p. 86)</p> <p>*59 glass bow in left hand battuto, then glissando bow on string as indicated. (p. 87)</p> <div data-bbox="940 1244 996 1316"> </div> <p>*60 battuto tremolo between fingers as indicated in score - numerals correspond as shown here . (p. 88)</p>	<p>*61 thumb roll, rosin on thumb will help to produce friction. (p. 88)</p> <p>*62 turning cranks so that string pitch goes higher or lower as indicated. (p. 88)</p> <p>*63 rub tension screw on string above nut as indicated, with pressure indicated; an arrow to R = towards the nut; an arrow to the left = towards the bridge. (p. 89)</p> <p>*64 rub tension screw on string ord., “long”, “mid” and “short” refer to relative length of rub stroke along string. (p. 89)</p> <p>*65 play as fast as possible on all four strings, in a random order. (p. 90)</p> <p>*66 harmonic glissando – natural harmonics are found at several locations along the string, so that as the finger moves towards the bridge or nut, a portion of the harmonic series with the greatest amplitude (for example #2-8) may repeat several times over a one or two octave glissando cycle. (p. 93)</p> <p>*67 finger slide oscillation between the indicated harmonics, keeping the finger on the string at all times. (p. 94)</p> <p>*68 dual harmonic glissando, similar to #66 and performed on adjacent strings. (p. 94)</p> <p>*69 oscillation between harmonic and open string. (p. 95)</p> <p>*70 alternate fingers. (p. 95)</p> <p>*71 alternate fingers and strings. (p. 95)</p> <p>*72 oscillate between harmonic and open string, while sustaining pitch on adjacent string. (p. 95)</p> <p>*73 artificial and natural harmonics sounded together. (p. 95)</p> <p>*74 tremolo between harmonic glissando and open string. (p. 96)</p> <p>*75 harmonic pizzicato. (p. 96)</p> <p>*76 pulled harmonic – finger touching node on the inside of the string and gently pull string outward, causing a slight microtonal rise in pitch, generally no more than a ½ step higher. (p. 97)</p> <p>*77 left-hand pizzicato with right-hand harmonic. (p. 97)</p> <p>*78 violin 1 remains at fast tempo and rejoins ensemble in mm 183, beat 3. (p. 98)</p> <p>*79 compound harmonic on one string – this refers to adding a harmonic on top of an already produced artificial harmonic (3 elements: 1. Stop; 2. Harmonic (usually at P4); 3. A second harmonic at interval specified above (again usually a P4). The result is an unstable sonority and temporally shifting depending greatly on the stability of the first harmonic. (p. 104)</p> <p>*80 as high as possible, moving hand near bow – tone may disappear. (p. 104)</p> <p>*81 bow on strings as indicated, chord is important as it effects resonance even when not explicitly initiated. (p. 104)</p> <p>*82 less expensive bow (with normal-to slightly increased hair tension) is suggested so that stand battuto may be performed with a wide dynamic range and variance of expression. (p. 107)</p> <p>*83 thrash bow through air. (p. 109)</p> <p>*84 violin 2 radically detune all strings as indicated, retaining enough tension to produce pitch – all three instruments will reduce bow pressure to 0% at the end. (p. 116)</p>
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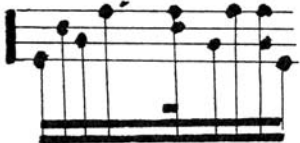
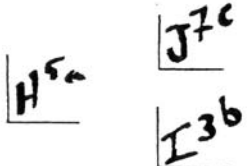
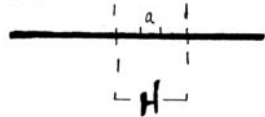
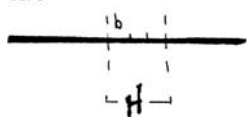
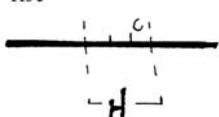

a.7040				6624	48	7200	32					7200	16		7048,8	24	7040	16	7150	11			6864	24	6880	16							
g#.6644,8												6750	15				6600	15	6500	10					6450	15	6700	10					
g.6272	6048	92	6272	64					6080	32					6208	32						6304	32							G			
f#.5920											6000	20	6300	14				6160	14	5850	9					6020	14	6030	9	F			
f.5586											5700	19	5850	13				5720	13							5590	13						
e.5274						5400	24				5400	18	5400	12				5280	12	5200	8			5148	18	5160	12	5360	8	E			
d#.4978											5100	17												4862	17								
d.4698,6			4704	48	4416	32				4560	24	4800	16	4950	11	4656	24	4699,2	16	4840	11	4550	7	4728	24	4576	16	4730	11	4690	7	D	
c#.4435												4500	15	4500	10			4405,5	15	4400	10				4290	15	4300	10					
c.4186	4032	64							4185,6	32																					C		
b.3951,1											4200	14	4050	9	3880	20	4111,8	14	3460	9	3900	6			4004	14	3870	9	4020	6	B		
a#.3729,3											3900	13			3686	19	3818,1	13							3718	13							
a.3520				3312	24	3600	16				3600	12	3600	8	3492	18	3524,4	12	3520	8					3432	12	3440	8			A		
g#.3322,4						3375	15										3298	17		3250	5			3146	11		3010	7	3350	5			
g.3136	3024	48	3136	32				3139,2	24	3040	16	3300	11	3150	7	3104	16	3230,7	11	3080	7			3152	16						G		
f#.2960						3150	14			2850	15	3000	10			2910	15	2937	10					2955	15	2860	10						
f.2793						2925	13																								F		
e.2637						2700	12			2660	14			2716	14			2643,3	9	2640	6	2600	4	2758	14		2574	9	2580	6	2680	4	E
d#.2489										2470	13					2522	13							2561	13								
d.2349,3			2352	24	2208	16	2475	11		2280	12	2400	8					2349,6	8					2364	12	2288	8					D	
c#.2217,5						2250	10						2250	5					2200	5													
c.2093	2016	32							2092,8	16	2090	11	2100	7		2134	11	2055,9	7					2167	11	2002	7					C	
b.1975,5	1953	31	1960	20		2025	9	1962	15	1900	10				1940	10				1950	3	1970	10						2010	3		B	
a#.1864,7	1890	30	1862	19																													
a.1760	1827	29			1656	12	1800	8	1831,2	14	1710	9	1800	6	1800	4	1746	9	1762,2	6	1760	4		1773	9	1716	6	1720	4			A	
g#.1661,2	1764	28	1764	18																													
	1701	27			1518	11	1575	7	1700,4	13																							
	1638	26	1666	17																													
	1575	25																															
g.1568	1512	24	1568	16					1569,6	12	1520	8			1552	8							1576	8								G	
f#.1480	1449	23	1470	15	1380	10					1500	5					1468,5	5						1379	7		1430	5					
f.1396,9	1386	22						1438,8	11	1330	7					1358	7															F	
	1323	21	1372	14																													
e.1318,5	1260	20			1242	9	1350	6	1308	10			1350	3					1320	3	1300	2						1290	3	1340	2	E	
d#.1244,5	1197	19	1274	13																													
d.1174,7	1134	18	1176	12	1104	8			1177,2	9	1140	6	1200	4		1164	6	1179,8	4					1182	6	1144	4					D	
c#.1108,7	1071	17					1125	5																									
c.1046,5	1008	16	1078	11	966	7				1046,4	8																					C	
b.987,77	945	15	980	10						950	5					970	5							985	5							B	

a#.932,33	882	14					915,6	7																				
a.880	819	13	882	9	828	6	900	4			900	3	900	2		881,1	3	880	2			858	3	860	2			A
g#.830,61																												
g.783,99	756	12	784	8						784,8	6	760	4			776	4					788	4					
f#.739,99					690	5																						
f.698,46	693	11	686	7																								
e.659,26	630	10				675	3	654	5									e.650-	1						e.670+	1		
d#.622,25																												
d.587,33	567	9	588	6	552	4				570	3	600	2			582	3	587,4	2			591	3	572	2			
c#.554,37																												
c.523,25	504	8						523,2	4																			
b.493,88			490	5																								
a#.466,16	441	7																										
a.440					414	3	450	2					a.450+	1				a.440=	1					a.430-	1			
415,30																												
g.392	378	6	392	4					392,4	3	380	2				388	2					394	2					
369,99																												
f.349,23																												
e.329,63	315	5																										
d#.311,13																												
d.293,67			294	3	276	2						d.300+	1					d.293,7=	1					d.286-	1			
c#.277,18																												
c.261,63	252	4						261,4	2																			
b.246,94																												
a#.233,08																												
a.220							a.225+	1																				
207,65																												
g.196	189	3	196	2								g.190-	1									g.198+	1					
185																												
f.174,61																												
e.164,81																												
d#.155,56																												
d.146,83						d.138,-	1																					
c#.138,59																												
c.130,81	126	2						c.130,8=	1																			
b.123,47																												
a#.116,54																												
a.110																												
103,83																												
g.97,99						g.98,=	1																					
92,49																												

f.87,30																	
e.82,40																	
d#. 77.78																	
d.73,41																	
c#. 69.29																	
c.65,40	c.63,- 1																
B.61,73																	
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	C	G	D	A	C	G	D	A	G	D	A	E	G	D	A	E	
	cello-----				viola-----				violin 2-----				violin 1-----				

Ideally, all movements will be performed as a single unit in concert. However, as time and resources of ensembles are limited, single movements (or any combination thereof) may be performed in concert. The relative timings of each movement are as follows:	
I.	17’33’’
II.	9’03’’
III.	6’01’’
IV.	6’03’’
V.	20’52’’
Total Length:	59’32’’

Addendum

 	<p>Rhythm of bow on strings, usually in conjunction with ascending or descending chord</p> <p>Regions H, I, J are approximately 1/3 of regions A, B or C – or approximately 1/9 of bow. These regions consist of a larger area than a normal tremolo. At times, it is desired to work within different sub-units within the larger A, B or C regions. Therefore regions H, I and J will feature three different spatial orientations in order to feature micro-movement within the larger region. So that region H5a, consists of a region H, at angle 5 (ord), in the middle 3rd of the larger A region. Or as notated below.</p>
	<p>Examples:</p> <div data-bbox="577 810 840 954"><p>H5a</p></div> <div data-bbox="1099 810 1344 954"><p>H5b</p></div> <div data-bbox="1727 810 1944 954"><p>H5c</p></div>
	<p>Bow pressure is separately identified, when necessary to offset effect of normal bow pressure; such as when using high bow speeds.</p>